ST565 Midterm

Winter 2016

Answer the questions in the spaces provided on this exam.

Name: SOLUTIONS ____________________

- You have 80 minutes to complete the exam.
- There are 6 questions. Answer all of the questions.
- Please
  - do not look at the exam until I tell you and
  - stop writing when I announce that the exam is over.
- You may assume Z_t always refers to a white noise process with mean zero and variance σ^2.

Question	Points	Score
1	8	
2	6	
3	6	
4	8	
5	6	
6	6	
Total:	40	

1. (a) Define weak stationarity. (4)

Solution: A time series, X_t is weakly stationary if its mean and autocovariance functions do not depend on time. That is, $E(X_t)$ and $Cov(X_t, X_{t+h})$ do not depend on t.

(b) Give an example of a non-stationary process. (Make sure to state the property of the process that varies through time) (2)

Solution: $X_t = t + Z_t$, with a mean that depends on t. (Many alternatives here)

(c) Why is stationarity a desirable property for a time series process? (2)

Solution: With a stationary process, a longer time series (i.e. larger n) gives more information about the statistical properties (mean, autocorrelation, etc.) of the series.

Exam continues on the next page ...

2. Consider the process $x_t = x_{t-1} + \frac{1}{4}Z_{t-1} + Z_t$, where Z_t is a white noise process.

   (a) Is the process invertible? (2)

Solution: Yes, invertible since root of MA polynomial is outside unit circle.

$$(1-B)x_t = (1+\tfrac{1}{4}B)Z_t \quad \text{in backshift form} \tag{1}$$
$$\text{MA polynomial:} \quad \theta(B) = 1 + \tfrac{1}{4}B \tag{2}$$
$$\text{Root of } \theta(B): \quad \tfrac{1}{4}B = -1 \implies B = -4 \tag{3}$$
$$\text{and} |-4| > 1 \tag{4}$$

   (b) Is the process stationary? (2)

Solution: No. Root of AR polynomial is not outside unit circle.

$$(1-B)x_t = (1+\tfrac{1}{4}B)Z_t \quad \text{in backshift form} \tag{5}$$
$$\text{AR polynomial:} \quad \phi(B) = 1 - B \tag{6}$$
$$\text{Root of } \phi(B): \quad B = 1 \tag{7}$$
$$\text{and } 1 \not> 1 \tag{8}$$

   (c) Is the first difference of the process, $y_t = x_t - x_{t-1}$, stationary? (2)

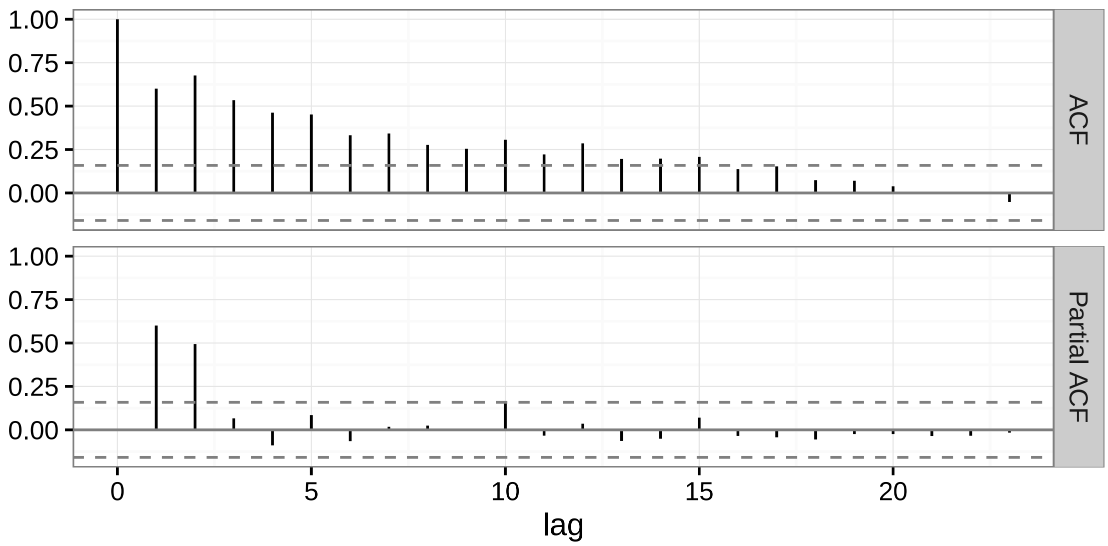
Solution: Yes.

$$(1-B)x_t = (1+\tfrac{1}{4}B)Z_t \quad \text{in backshift form} \tag{9}$$
$$\text{Notice:} \quad y_t = x_t - x_{t-1} = (1-B)x_t = (1+\tfrac{1}{4}B)Z_t \tag{10}$$

...which is an MA process, and thus stationary.

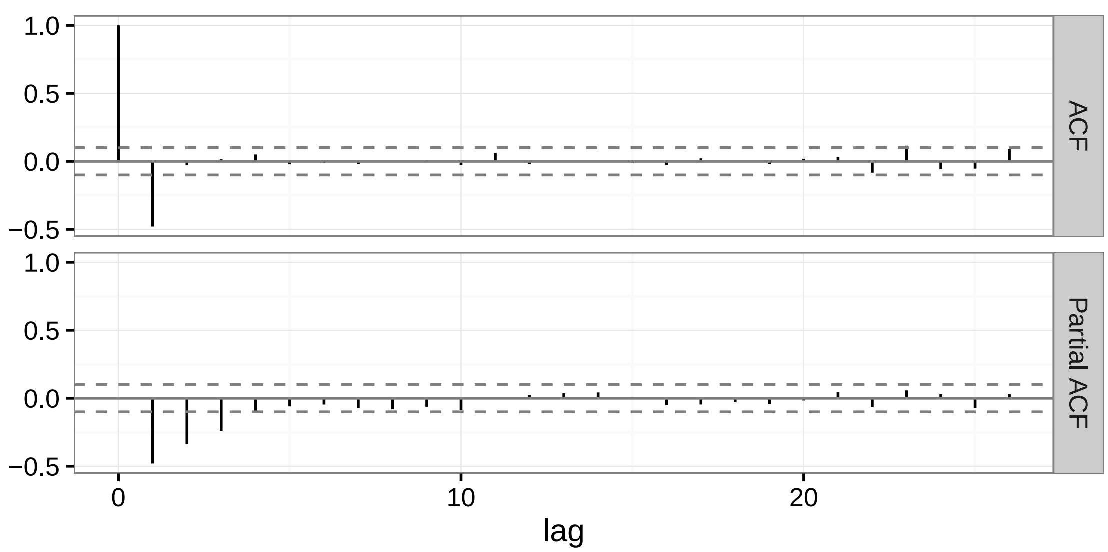
3. Below are the ACF and PACF for three time series. For each series, state whether it is autoregressive or moving average, and the order (p or q).

   (a) AR or MA (circle one), $p =$ _____ , $q =$ _____? (2)



Solution: AR(2), q = 0.

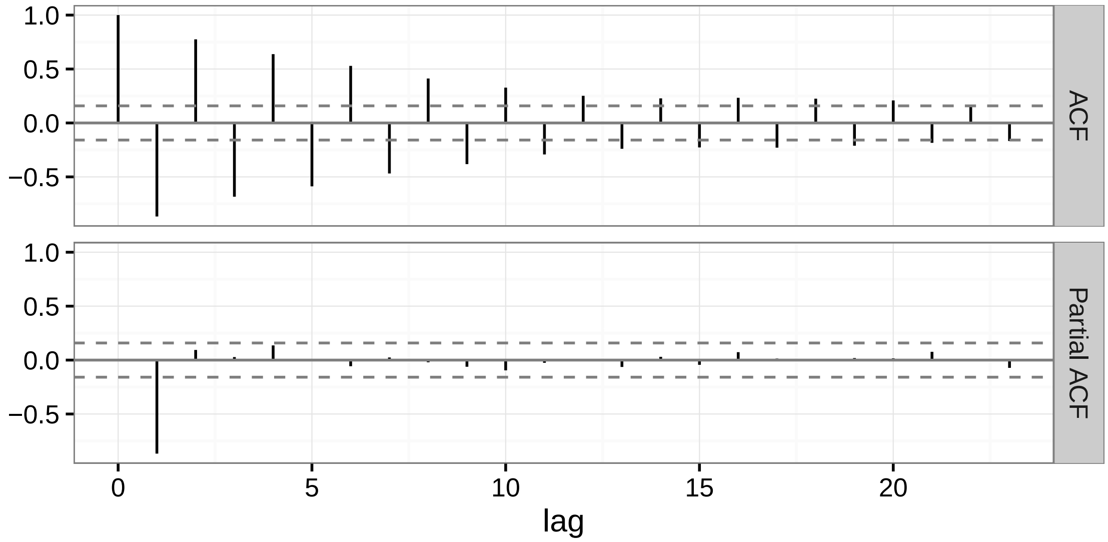
   (b) AR or MA (circle one), $p =$ _____ , $q =$ _____? (2)



Solution: MA(1), p = 0.

Question 3 continues on the next page…

(c) AR or MA (circle one), $p =$ ______ , $q =$ ______? (2)



Solution: AR(1), q = 0.

4. (a) Describe the difference between the *empirical autocorrelation function*, $r(h)$, (also known as *sample autocorrelation function*), and the *theoretical autocorrelation function*, $\rho(h)$. (4)

> **Solution:** The empirical ACF is calculated from the data, or the sample observed, while the theoretical ACF is mathematically derived from the defined form of a process.

(b) How are the empirical autocorrelation function and theoretical autocorrelation function used to identify a time series model? (4)

> **Solution:** Essentially, the empirical ACF is compared to theoretical ACFs, we choose a process for our time series that has a theoretical ACF consistent with the sample ACF observed.

5. ARIMA models include a parameter, d, that controls the number of times a time series is differenced before being modeled by an ARMA process.

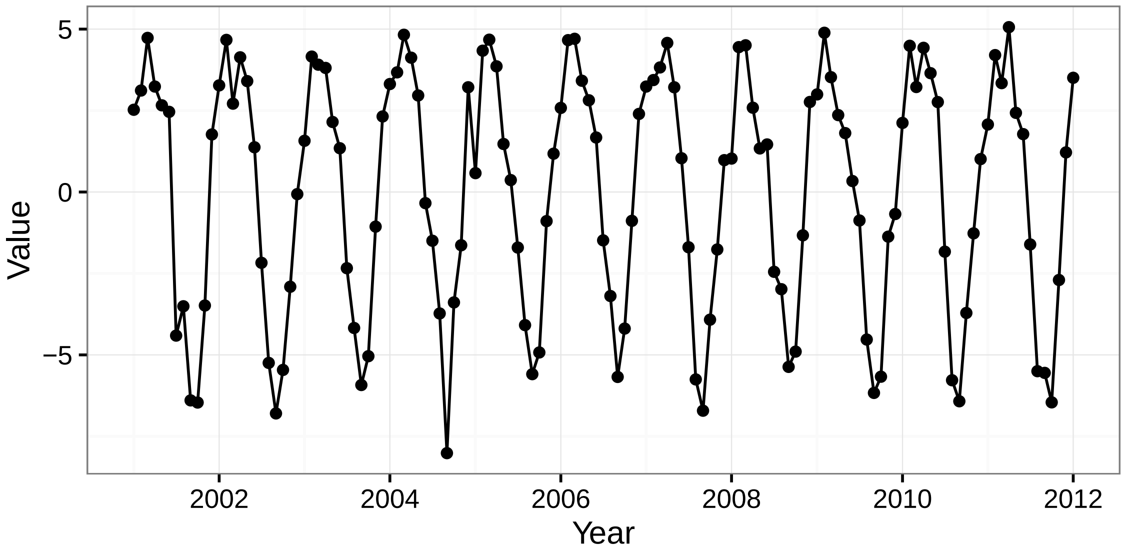
   (a) Why is differencing a time series sometimes necessary? (3)

   Solution: To achieve stationarity. (Remember: differencing removes non-stationarity beyond just trend, e.g. differencing a random walk removes the non-stationary variance)

   (b) How could you choose the amount of differencing required for a particular time series? (3)

   Solution: Observe the series after each difference, and difference until it appears stationary.

6. Below is a plot of a monthly time series. The investigator who collected it expects there to be an annual seasonal pattern, but she is more interested in the long term trend.

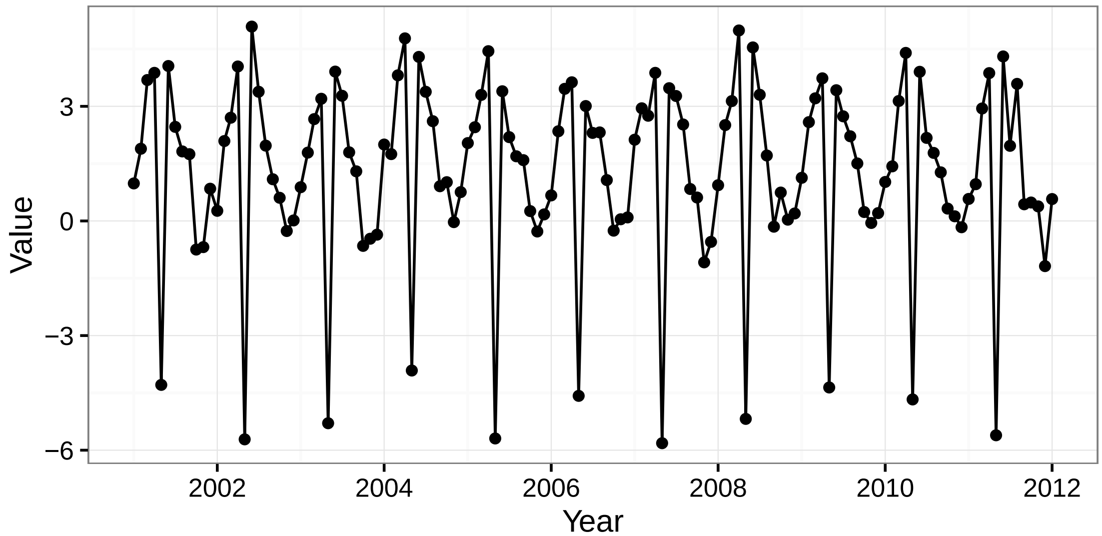


(a) Suggest a method she could use to model the seasonality, along with sentence explaining your choice. (3)

> **Solution:** More than one correct answer. A couple of correct answers are: (i) loess regression of value against month, and then model the residuals. (ii) calculate monthly means, and model deviations from those means. (Differencing seasonally will remove the trend too so that is not an ideal answer here)

 Question 6 continues on the next page…

(b) She brings you a second monthly time series shown below. (3)



Would you recommend the same method to model the seasonality in this time series? Give a one or two sentence explanation of your answer.

Solution: This situation is different. The same approach won't work, because now there is an irregularly timed, but extreme, drop within each cycle. Even further, that drop is for a single month, so smoothing is not longer reasonable. Removing monthly means should still work.