Stat 565

Spurious Regression

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Last time:

We can extend regression to deal with correlated errors.

Today:

Explore the concept of spurious correlation/regression

What is it?

How can we deal with it?

Practice with some time series regression analyses.

Your turn

Where does most of the variation in temperature come from?

What about particulates?

What about mortality?

Spurious correlation

With **completely independent** series:

Positive correlation can arise:

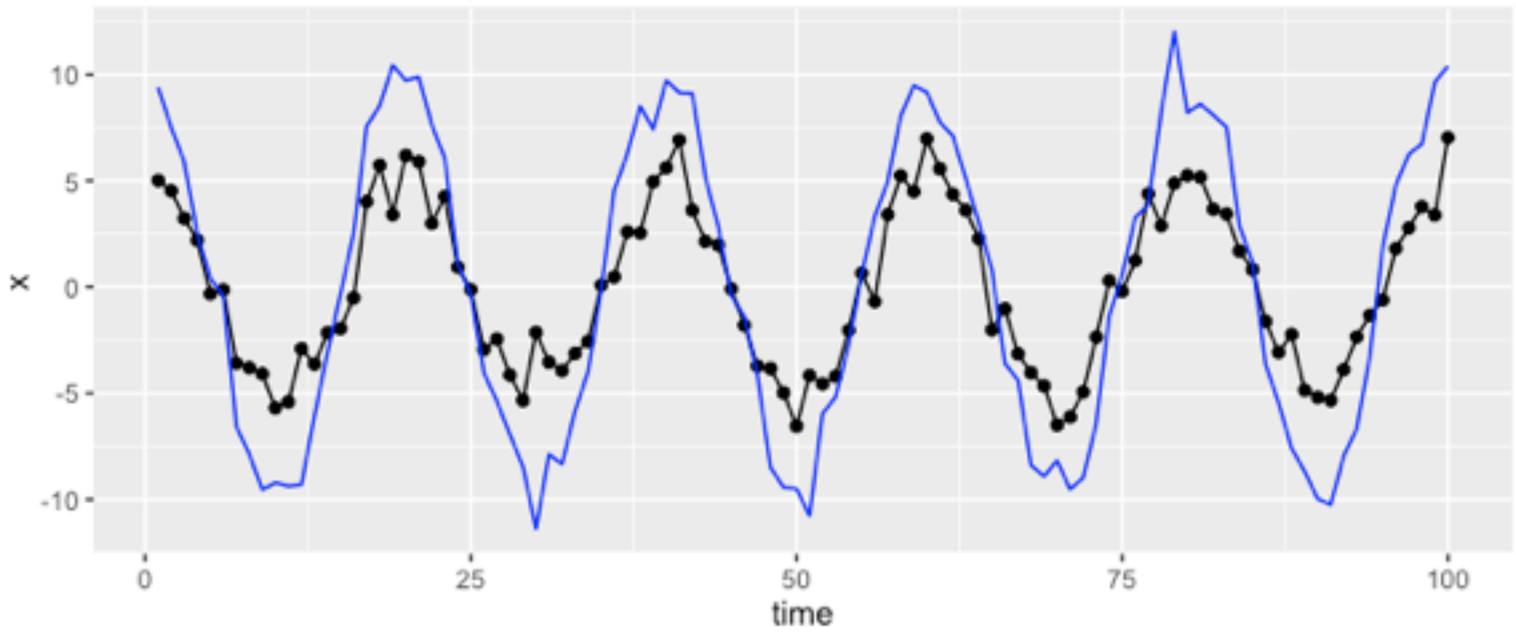
If trend dominates the series, and they are both trending in the same direction.

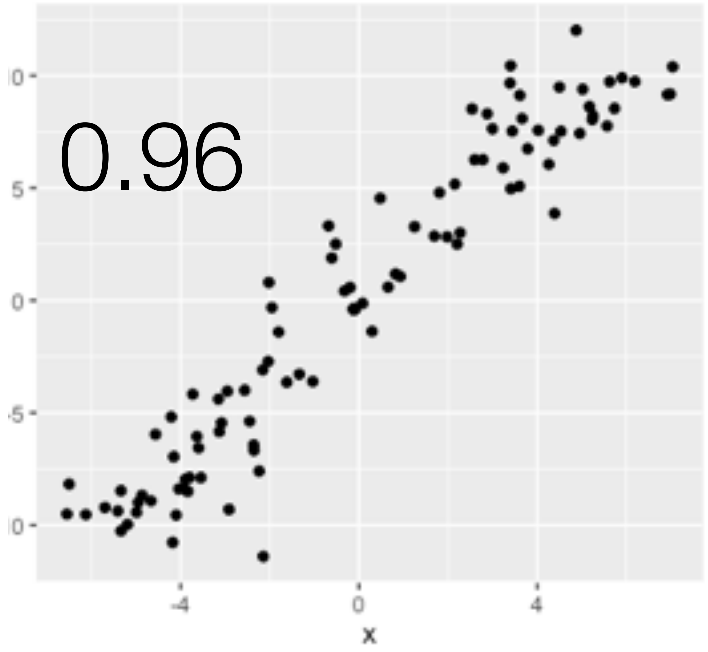
If seasonality dominates the series, the cycle length is roughly the same, and the cycles are in phase

If the cycles are out of phase the relationship will appear non-linear.

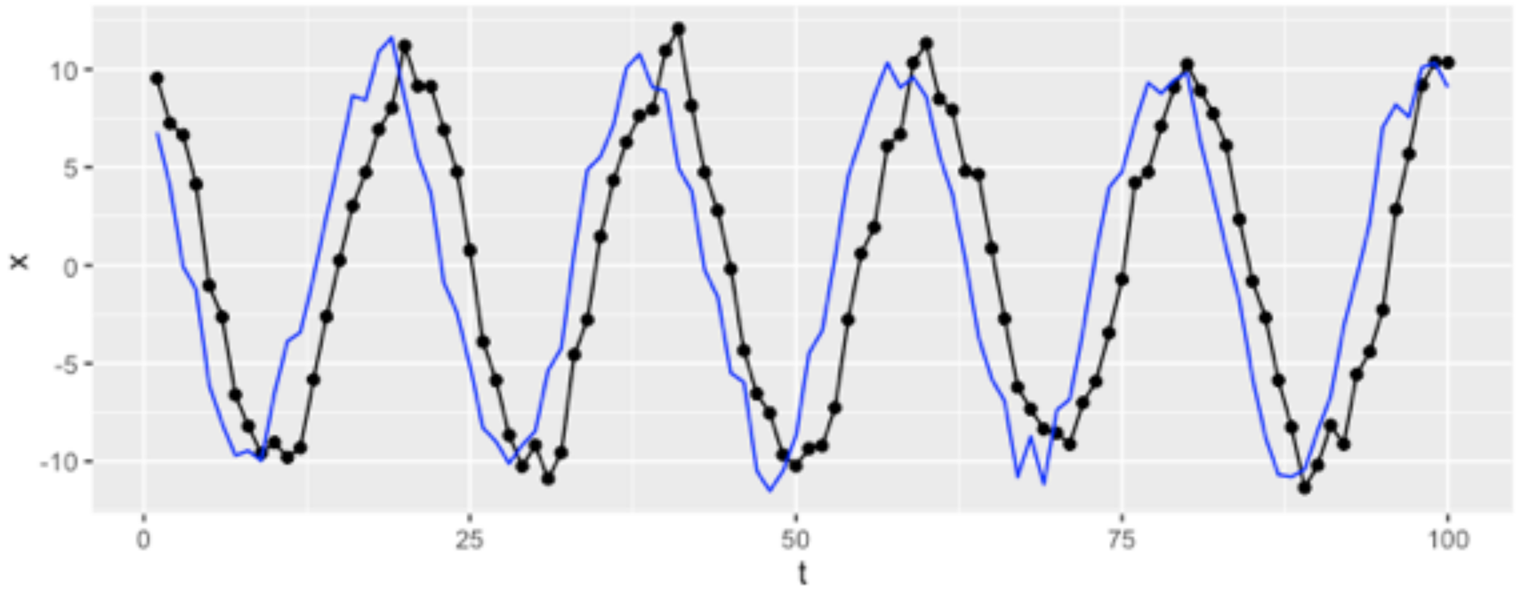
Simulated examples

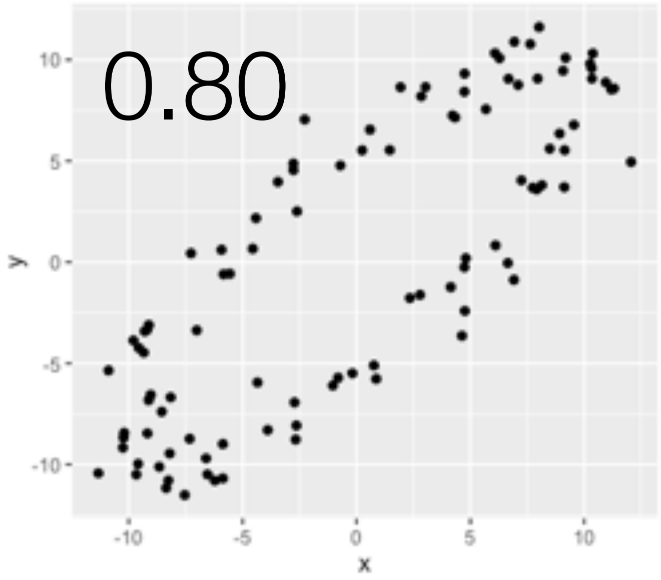
$x_t = 5 \cos(2\pi\, t / 10) + z_t$ $y_t = 10 \cos(2\pi\, t / 10) + w_t$




0.96

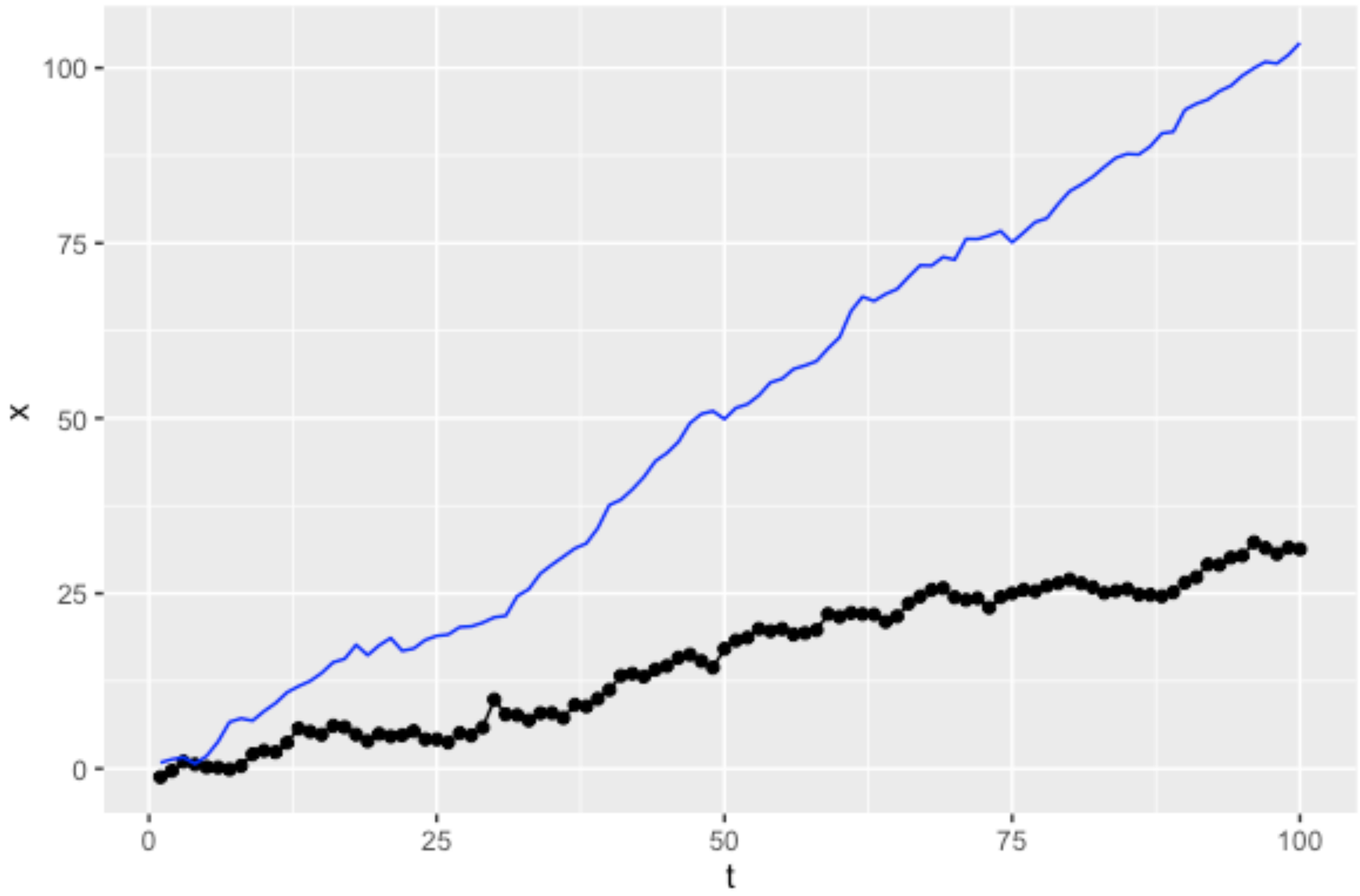
$x_t = 10 \cos(2\pi\, t / 10) + z_t$ $y_t = 10 \cos(2\pi\, (t+2) / 10) + w_t$

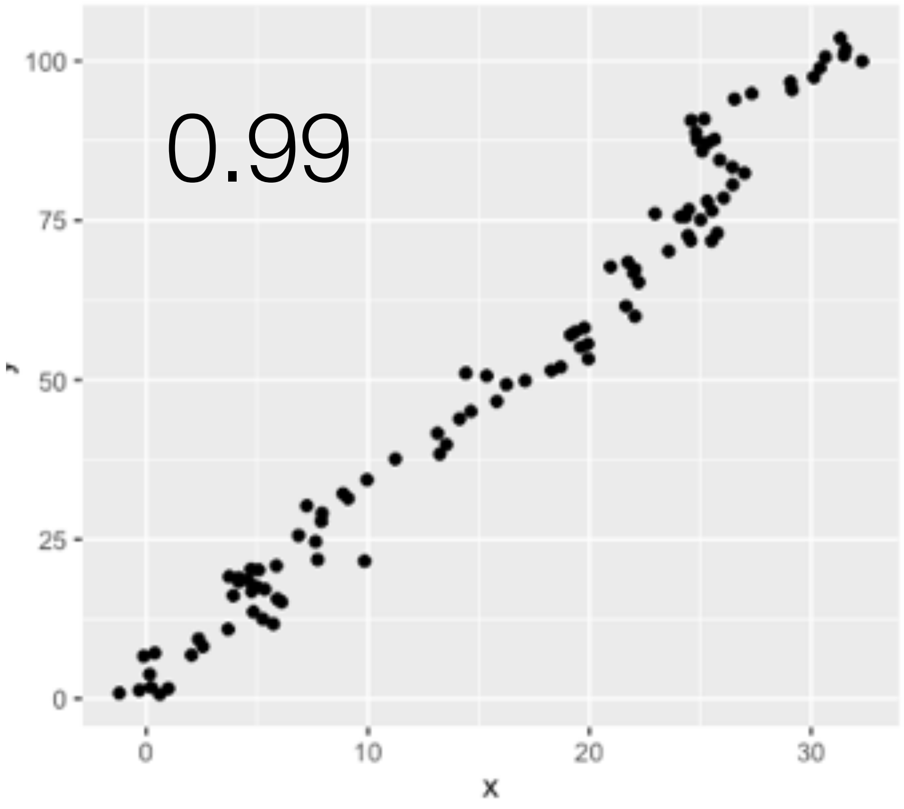



0.80

w_t, z_t independent white noise

Simulated examples





$x_t = 0.5 + x_{t-1} + z_t$ $y_t = 1 + y_{t-1} + w_t$

w_t, z_t independent white noise

Real world examples

Your turn

What do you think is driving the correlation?

Snake bites are positively associated with ice cream sales.

Electricity usage is positively associated with chocolate consumption in Australia over the last 50 years.

The stock market is positively correlated with sunspot activity.

What might be surprising…

Is that this problem also arises with stationary series...

Consider the regression model:

$$y_t = 1 + \beta x_t + z_t$$

where x_t is white noise and z_t is white noise.

Imagine, I observe 100 observations each of y_t and x_t,

and estimate the above model,

then test the null hypothesis $\beta = 0$.

If in reality $\beta = 0$, how often will I reject the null hypothesis at the 5% level?

```
one_sim <- function(){
  n <- 100
  x <- arima.sim(list(), n = n)
  z <- arima.sim(list(), n = n)
  y <- 1 + 0*x + z
  fit <- arima(y, order = c(0, 0, 0), xreg = x)

  abs( fit$coef[”x”] /
      sqrt(diag(fit$var.coef)[”x”]) ) > 1.96
}

reject <- replicate(1000, one_sim())
table(reject)
```

an empty list gives white noise

from Tues, fits a regression on x with ARMA errors

In pairs

http://stat565.cwick.co.nz/code/12-sim-code.r

But what if x_t or z_t are stationary time series?

Confirm:

- if **only one** of x or z is AR(1) the error rate is ~ 5%
- if **both** x and z are AR(1) the error rate is > 5%

How does the error rate depend on α? Fix α for x and vary α for z.

Conclusions, part 1

If one series is white noise, and the other is a correlated stationary time series, no problem.

If both series are correlated stationary time series, error rate is too high. We would tend to declare a relationship too often (“spurious regression”).

As the correlation increases error rate increases.

Examine the residuals for one fit when both x and z are AR(1) with parameter $\alpha = 0.9$.

Make a suggestion for fixing our regression.

Try it and see if the error rate is ~ 5%.

Conclusions, part 2

If both x and z are correlated stationary time series, we can get an error rate of ~5%, if we model the errors as a stationary time series.

You commonly see two approaches:

- Examine x, and choose an ARMA model. Use that model to “prewhiten” both x and y. Do OLS.
- Do OLS, examine residuals for an appropriate ARMA model. Do GLS.

How about stochastic trends?

We saw that as α increased, the error rate increased. If $\alpha=1$, we have a random walk.

Charlotte demonstrates that the error rate gets even worse.

But...differencing both series before regressing them solves the problem. This is equivalent to fitting an AR**I**MA model to the errors.

Things get really bad if you have an integrated random walk, error ~ 95%.

Strategy for regression

Difference to achieve stationarity first. Difference both series by the same amount, convention is use x to decide the differencing.

Fit regression model, use differenced series, or specify ARIMA(0, d, 0) errors.

Examine residuals to pick an ARMA model.

Fit regression model with ARIMA errors.

Check diagnostics.

Interpret.

Paired data analysis

Choose one from:

http://stat565.cwick.co.nz/code/12-pair-da.R

Bluebird chip prices and sales

Public transit boardings and gas prices

US personal income and consumption