(1)

Predict one step ahead SARIMA $(1,0,0)\times(0,1,1)_{12}$

$$x_t = x_{t-12} + \alpha\left(x_{t-1} - x_{t-13}\right) + Z_t + \beta Z_{t-12}$$

$$\underbrace{x_n^{n+1} = x_N(1)}_{\text{one step ahead forecast, given we have observed } n \text{ values.}} = E\left(x_{n+1} \mid x_n, x_{n-1}, \dots, x_1\right)$$

$$= E\left(x_{n-11} \mid x_n, \dots, x_1\right) + \alpha\Big(E\left(x_n \mid x_n \dots x_1\right) - E\left(x_{n-12} \mid x_n, \dots, x_1\right)\Big) +$$

$$E\left(Z_{n+1} \mid x_n, \dots, x_1\right) + \beta E\left(Z_{n-11} \mid x_n, \dots, x_1\right)$$

$$= x_{n-11} + \alpha\left(x_n - x_{n-12}\right) + 0 + \beta \hat{Z}_{n-11}$$

$$\left(x_{n-11} - \hat{x}_{n-11}\right)$$

residual at time n-4

observed value

expected value given ARMA model

$$x_n^{n+2} = x_{n-10} + \alpha\left(x_n^{n+1} - x_{n-11}\right) + \beta \hat{Z}_{n-10}$$

one step ahead prediction

AR(1) : $x_t = \alpha x_{t-1} + Z_t$

$$x_n^{n+1} = x_N(1) = E\left(\alpha x_n + Z_{n+1} \,\middle|\, x_n, \ldots, x_1\right)$$

$$= \alpha x_n$$

$$x_n^{n+2} = \alpha(\alpha x_n) = \alpha^2 x_n$$