Stat 565

(S)Arima & Forecasting

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Today

A note from HW #3

Pick up with AR**I**MA processes

Introduction to forecasting

HW #3

The sample autocorrelation coefficients are biased.
But asymptotically unbiased...

Theorem A.7 *If x_t is a stationary linear process of the form (1.31) satisfying the fourth moment condition (A.50), then for fixed K,*

$$\begin{pmatrix} \widehat{\rho}(1) \\ \vdots \\ \widehat{\rho}(K) \end{pmatrix} \sim AN\left[\begin{pmatrix} \rho(1) \\ \vdots \\ \rho(K) \end{pmatrix}, n^{-1}W\right],$$

where W is the matrix with elements given by

$$\begin{aligned} w_{pq} &= \sum_{u=-\infty}^{\infty}\Big[\rho(u+p)\rho(u+q) + \rho(u-p)\rho(u+q) + 2\rho(p)\rho(q)\rho^2(u) \\ &\qquad - 2\rho(p)\rho(u)\rho(u+q) - 2\rho(q)\rho(u)\rho(u+p)\Big] \\ &= \sum_{u=1}^{\infty}[\rho(u+p) + \rho(u-p) - 2\rho(p)\rho(u)] \\ &\qquad \times [\rho(u+q) + \rho(u-q) - 2\rho(q)\rho(u)], \end{aligned} \tag{A.55}$$

S&S

where the last form is more convenient.

For white noise, $W = I$,

and we have $r(h) \sim N(\rho(h), 1/n)$

Leads to CI's of the form $0 \pm 2/\sqrt{n}$ (the dashed lines in the acf plot).

HW #4 . . .

Simulation:

DO many times(

  simulate a process

  fit many AR models to the process

  find the AIC for each model

)

Suggestion:

do it once

wrap that in a function, i.e. write a function that does it for one series, fit_ars( )

replicate(1000, failwith(NA, fit_ars)())

One error will stop everything!

try, tryCatch in base R

dplyr::failwith()   failwith(NA, fit_ars)()

purrr::safely()

Or

method = “ML” in arima

Speed: microbenchmark package

HW #2 example

$$x_t = \beta_0 + \beta_1 t + w_t$$

a linear trend

$$\nabla x_t = x_t - x_{t-1} = \beta_1 + w_t - w_{t-1}$$

an MA(1) process with

constant mean β_1

Difference twice, that would remove a quadratic trend in t

x_t is called ARIMA(0, 1, 1)

ARIMA(p, d, q)

Autoregressive Integrated Moving Average

A process x_t is ARIMA(p, d, q) if x_t differenced d times ($\nabla^d x_t$) is an ARMA(p, q) process.

I.e. x_t is defined by

$$\phi(B) \nabla^d x_t = \theta(B) w_t$$

$$\phi(B) (1 - B)^d x_t = \theta(B) w_t$$

forces constant in 1st differenced series

```
arima(x, order = c(p, 1, q), xreg = 1:length(x))
```

Procedure for ARIMA modeling

We'll assume the primary goal is getting a forecast.

`diff`

1. Plot the data. Transform? Outliers? Differencing?
2. Difference until series is stationary, i.e. find d.
3. Examine differenced series and pick p and q.
4. Fit ARIMA(p, d, q) model to original data.
5. Check model diagnostics
6. Forecast (back transform?)

Pick one:

Oil prices

```
install.packages('TSA')
data(oil.price, package = 'TSA')
```

Global temperature

```
load(url("http://www.stat.pitt.edu/stoffer/tsa3/tsa3.rda"))
gtemp
```

US GNP

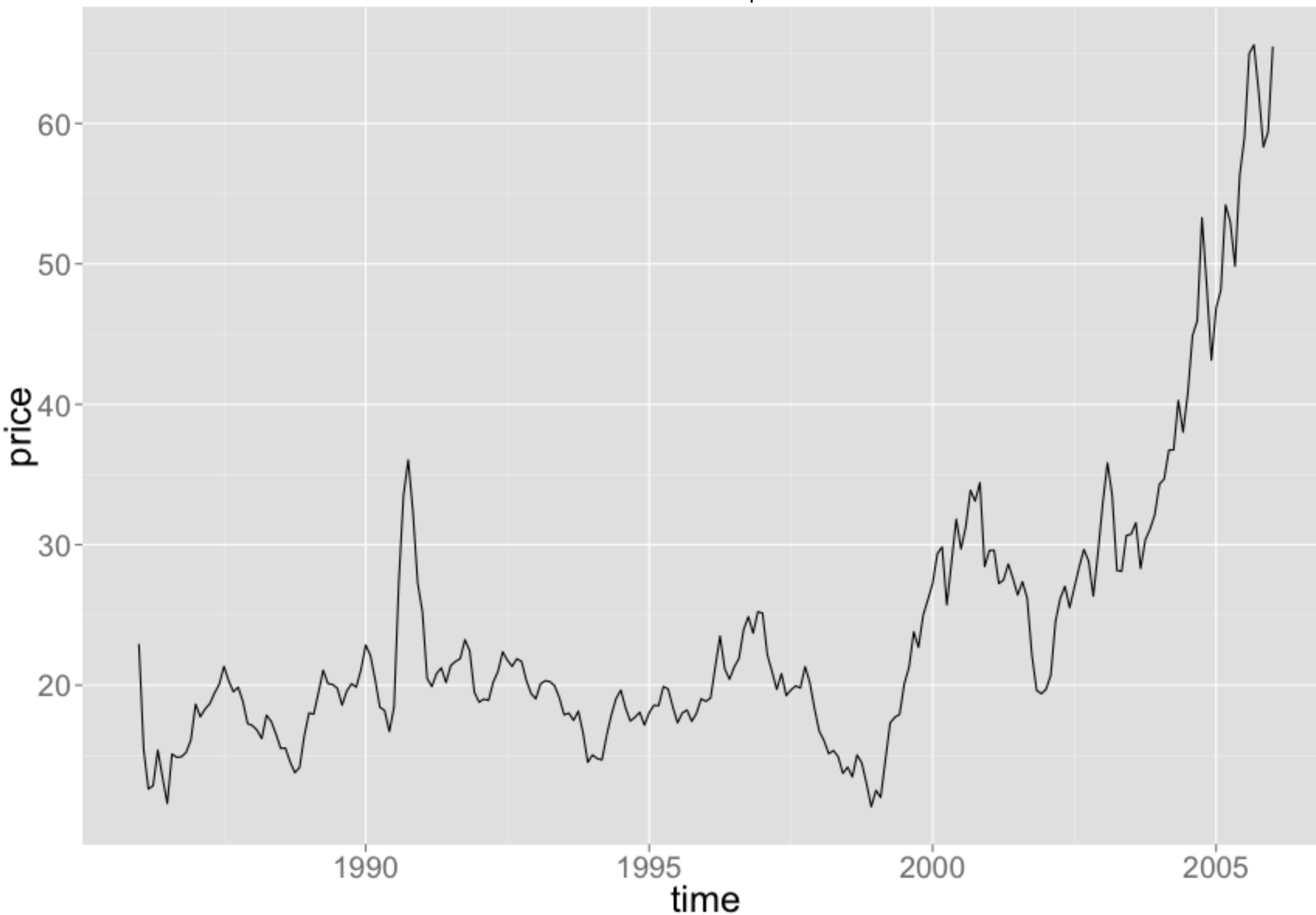
```
load(url("http://www.stat.pitt.edu/stoffer/tsa3/tsa3.rda"))
gnp
```

Sulphur Dioxide (LA county)

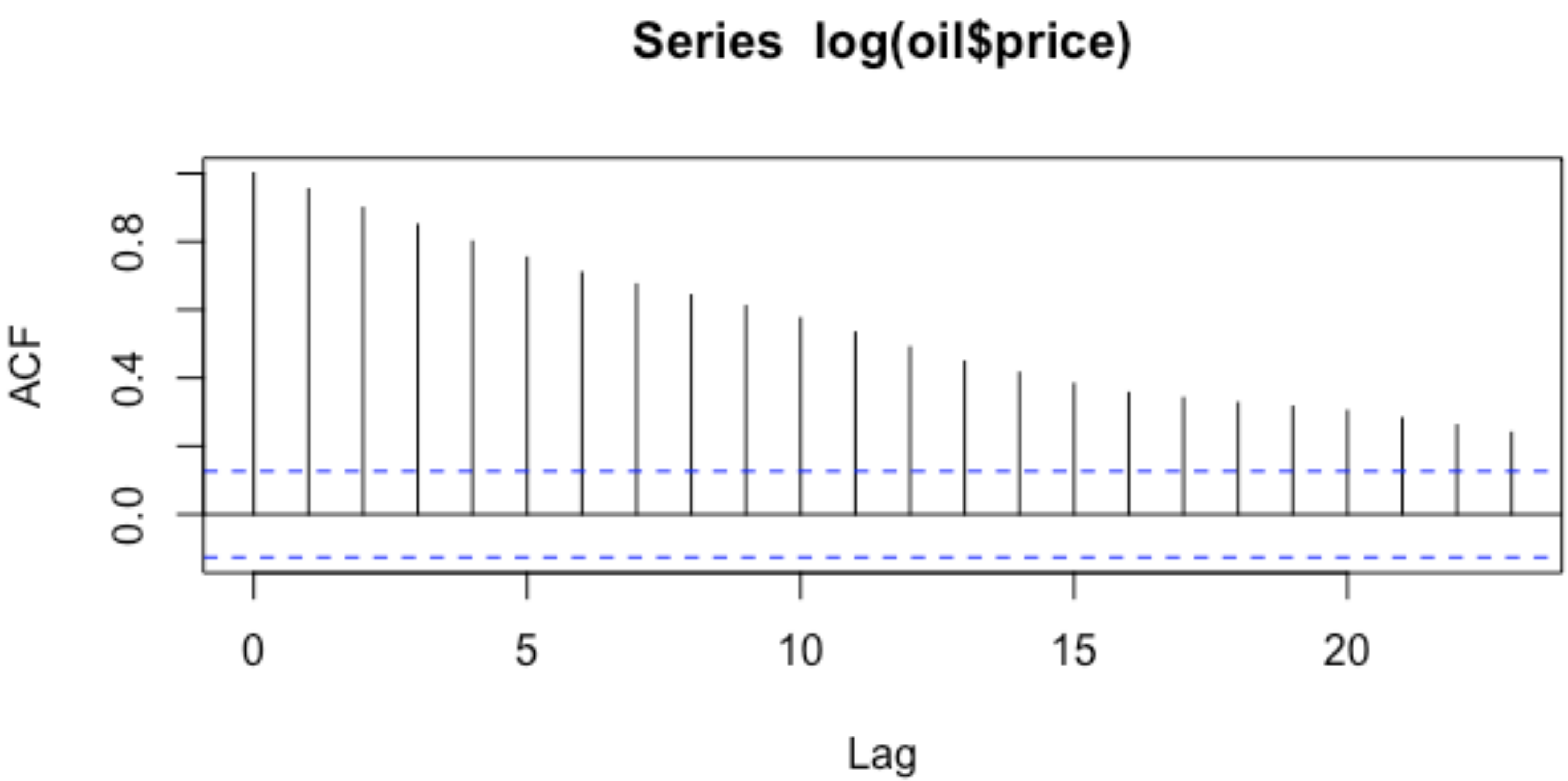
```
load(url("http://www.stat.pitt.edu/stoffer/tsa3/tsa3.rda"))
so2
```

1.

Ex 1 Oil prices

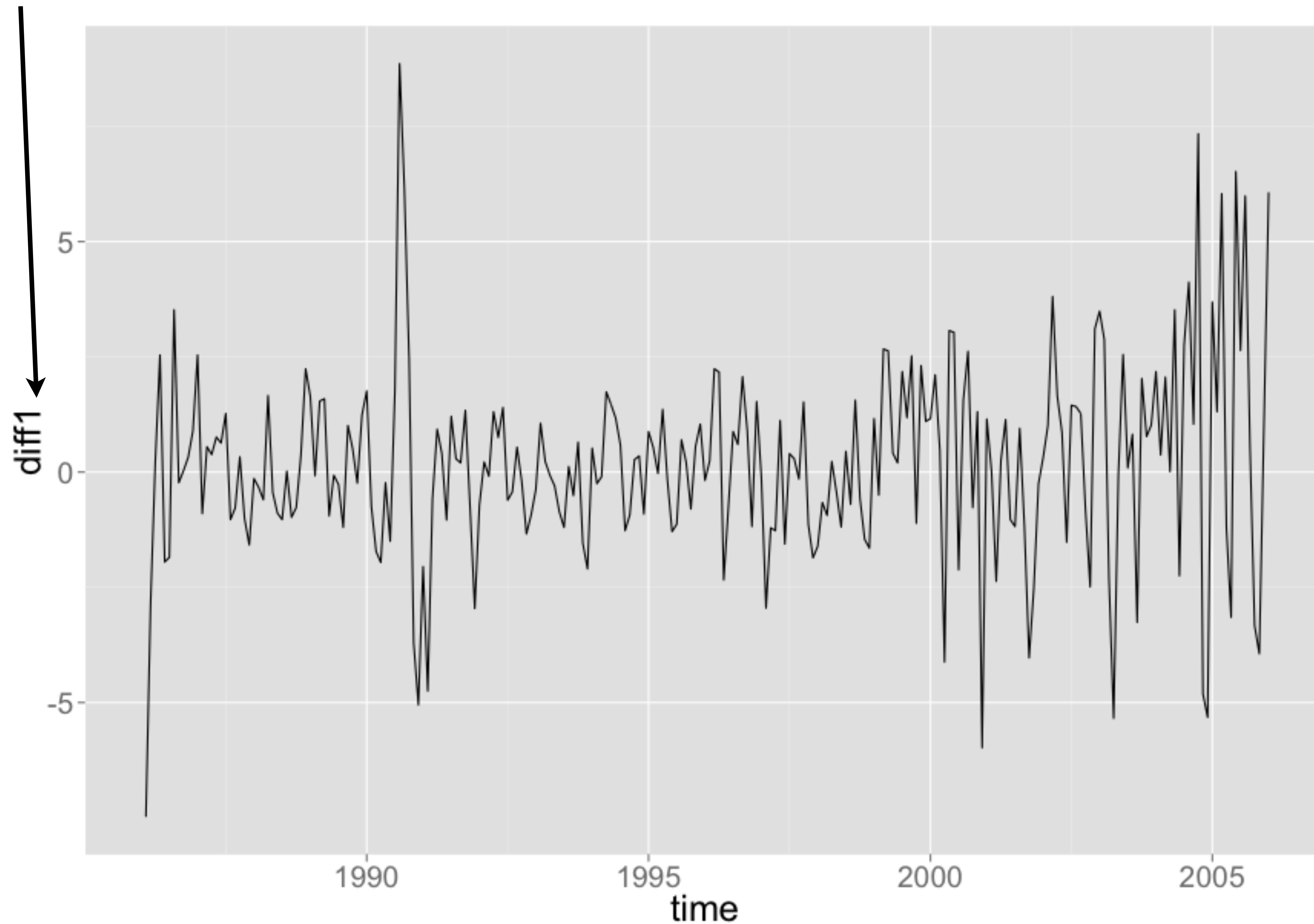


Linearly decreasing ACF, common sign of presence of trend, try differencing!

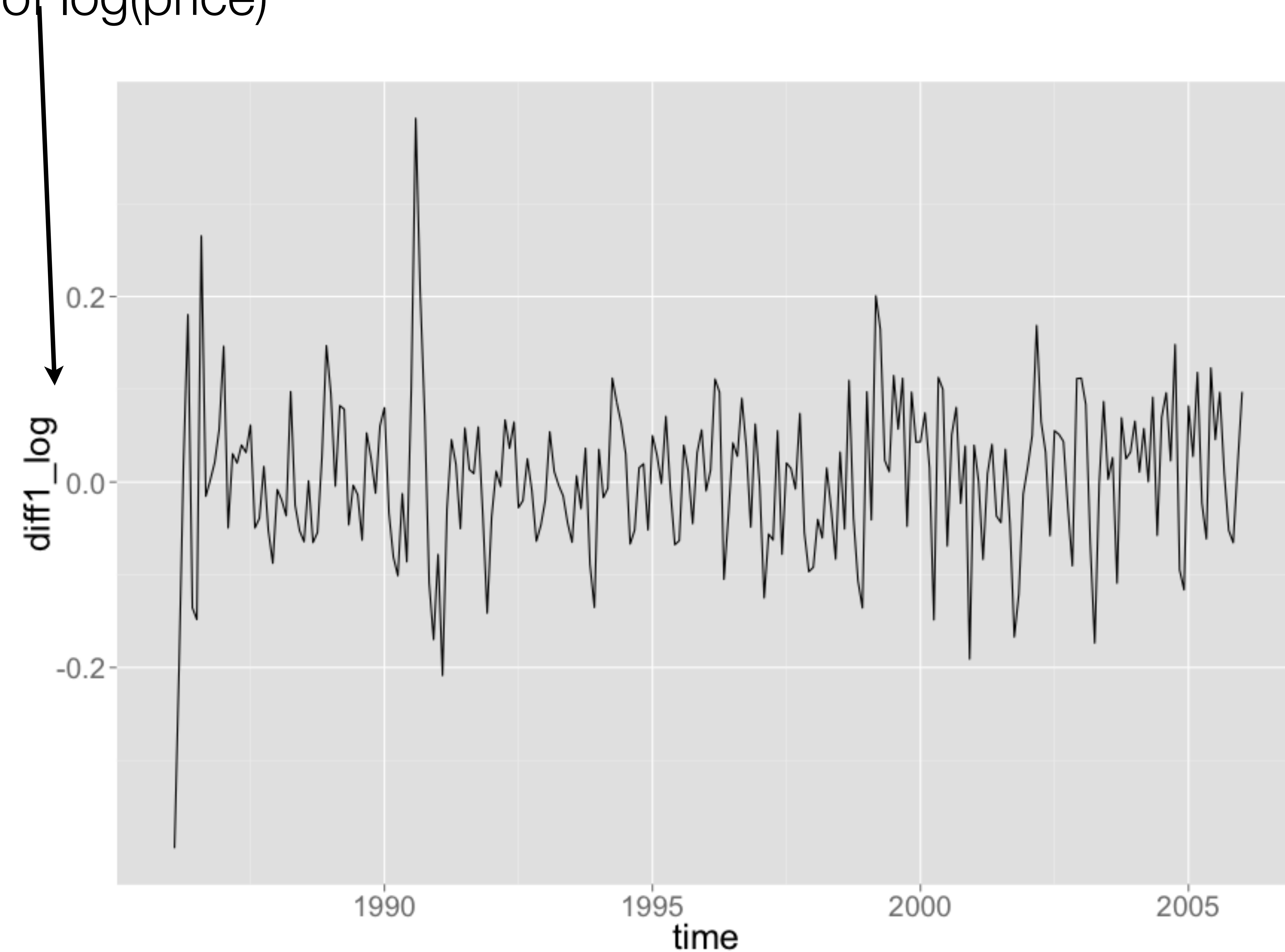


2.

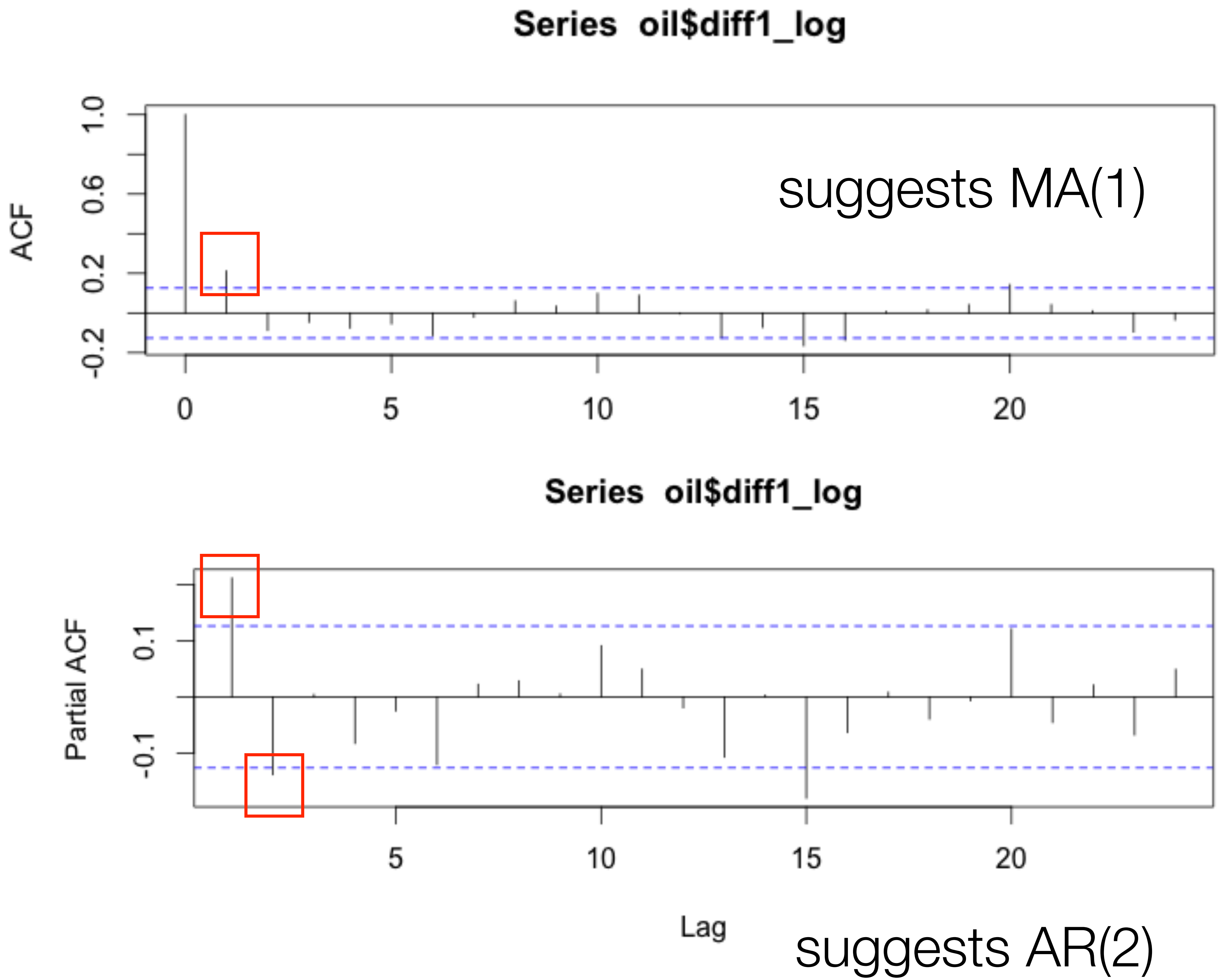
1st difference



2. 1st difference of log(price)



3. ACF and PACF on differenced log price



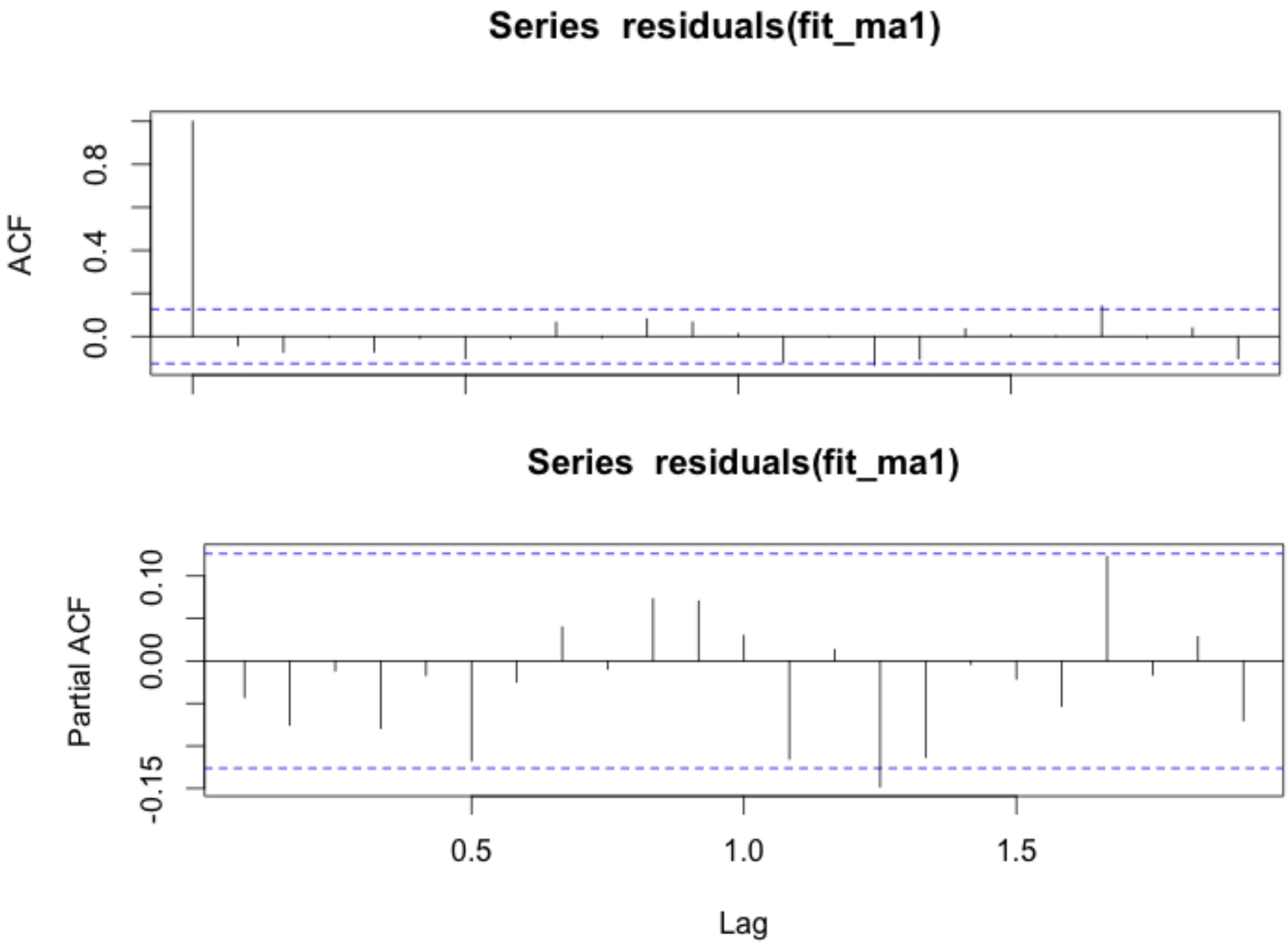
3.

```
n <- length(oil.price)
(fit_ma1 <- arima(log(oil.price), order = c(0, 1, 1), xreg = 1:n))
(fit_ar2 <- arima(log(oil.price), order = c(2, 1, 0), xreg = 1:n))
(fit_arma1 <- arima(log(oil.price), order = c(1, 1, 1), xreg = 1:n))
(fit_ma2 <- arima(log(oil.price), order = c(0, 1, 2), xreg = 1:n))
```

↑ trick ARIMA into estimating a constant in the differenced series

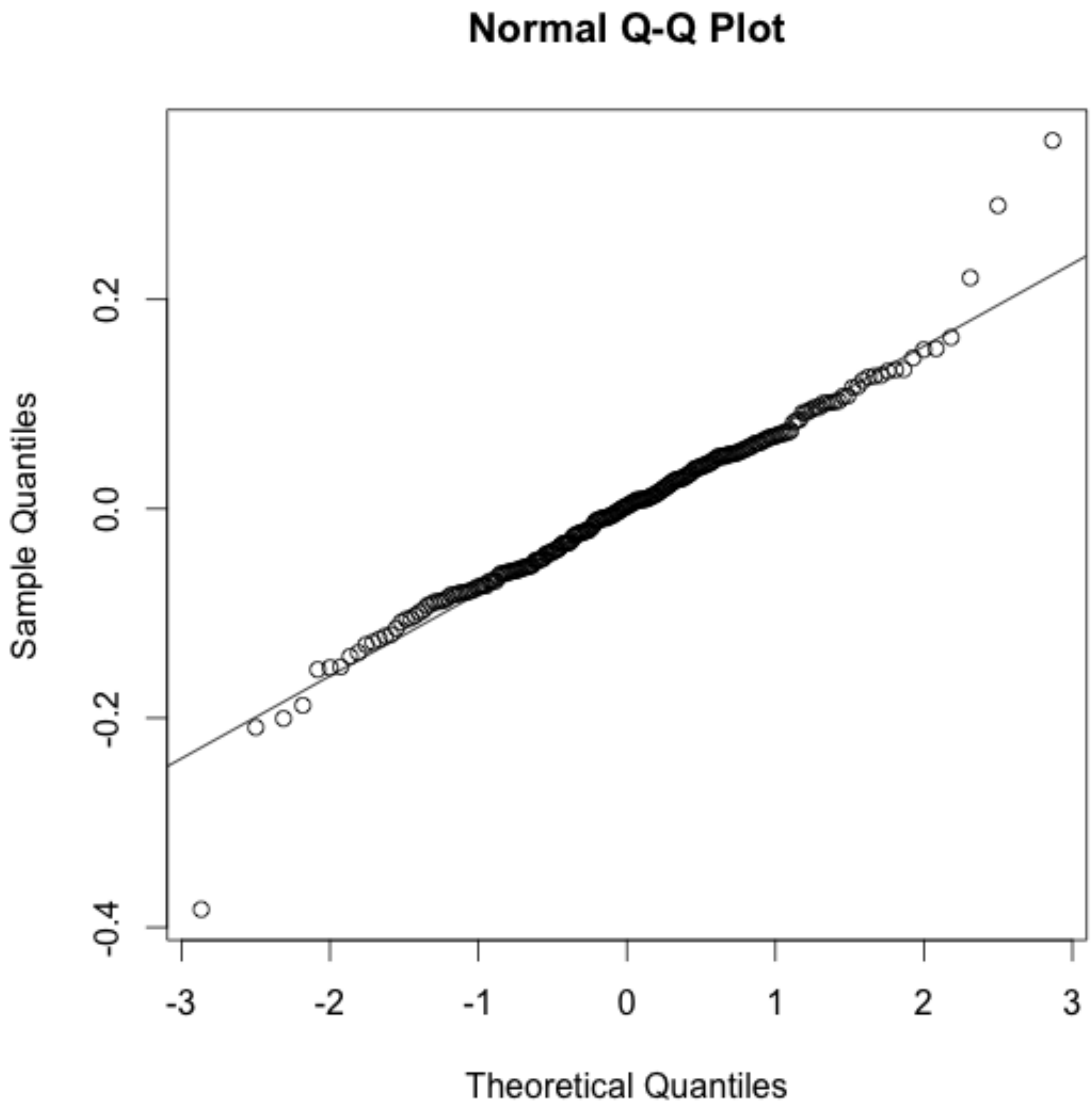
Choose MA(1) based on:
* smallest AIC
* in MA(2) θ_1 is roughly the same and θ_2 isn't significant.

4. ACF and PACF on residuals from MA(1) model

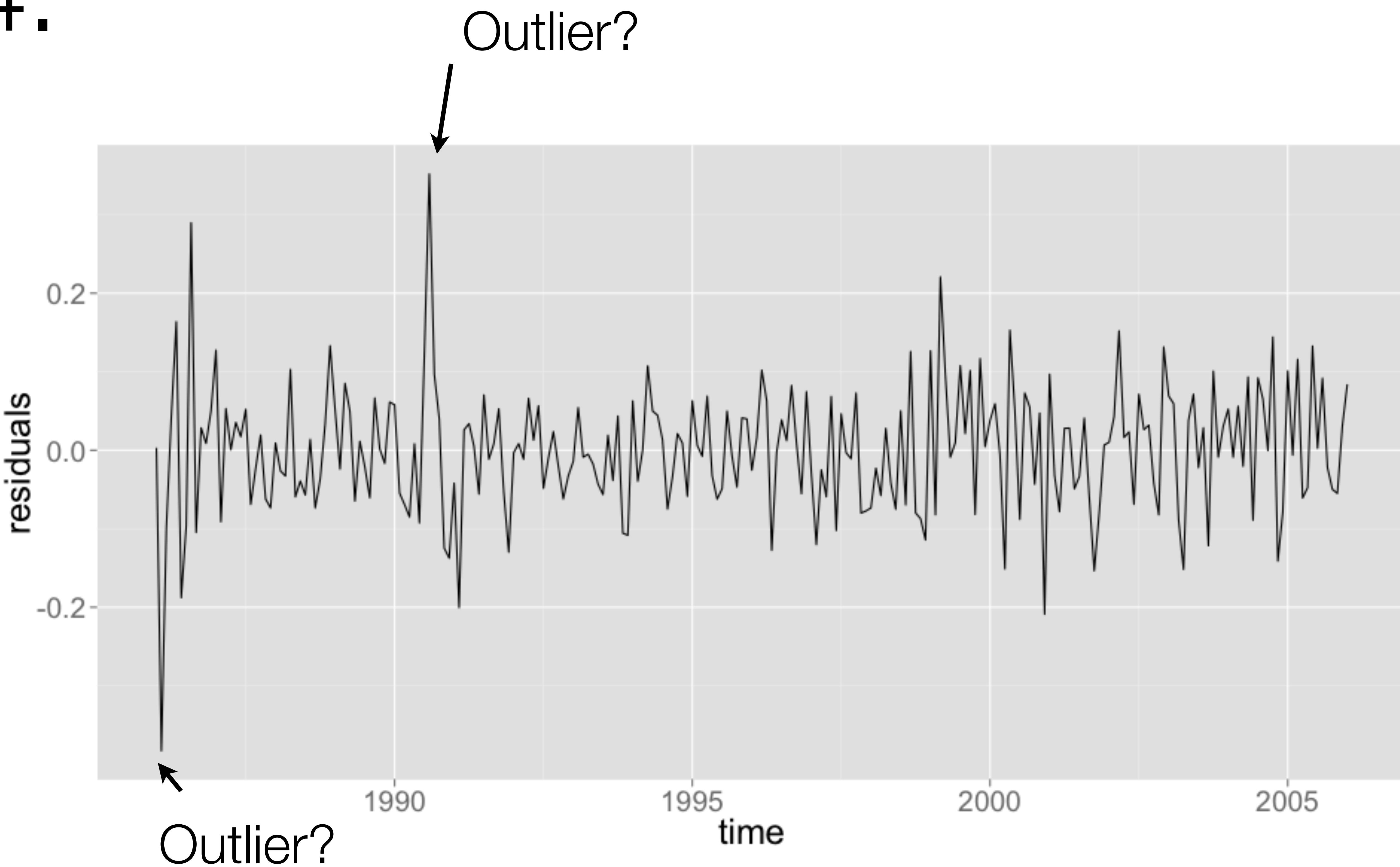


Look good!

4.


Normal Q-Q Plot
Sample Quantiles
Theoretical Quantiles
0.2
0.0
-0.2
-0.4
-3
-2
-1
0
1
2
3

4.



SARIMA models

I haven't shown you any data with seasonality.

The idea is very similar, if one seasonal cycle lasts for s measurements, then if we difference at lag s,

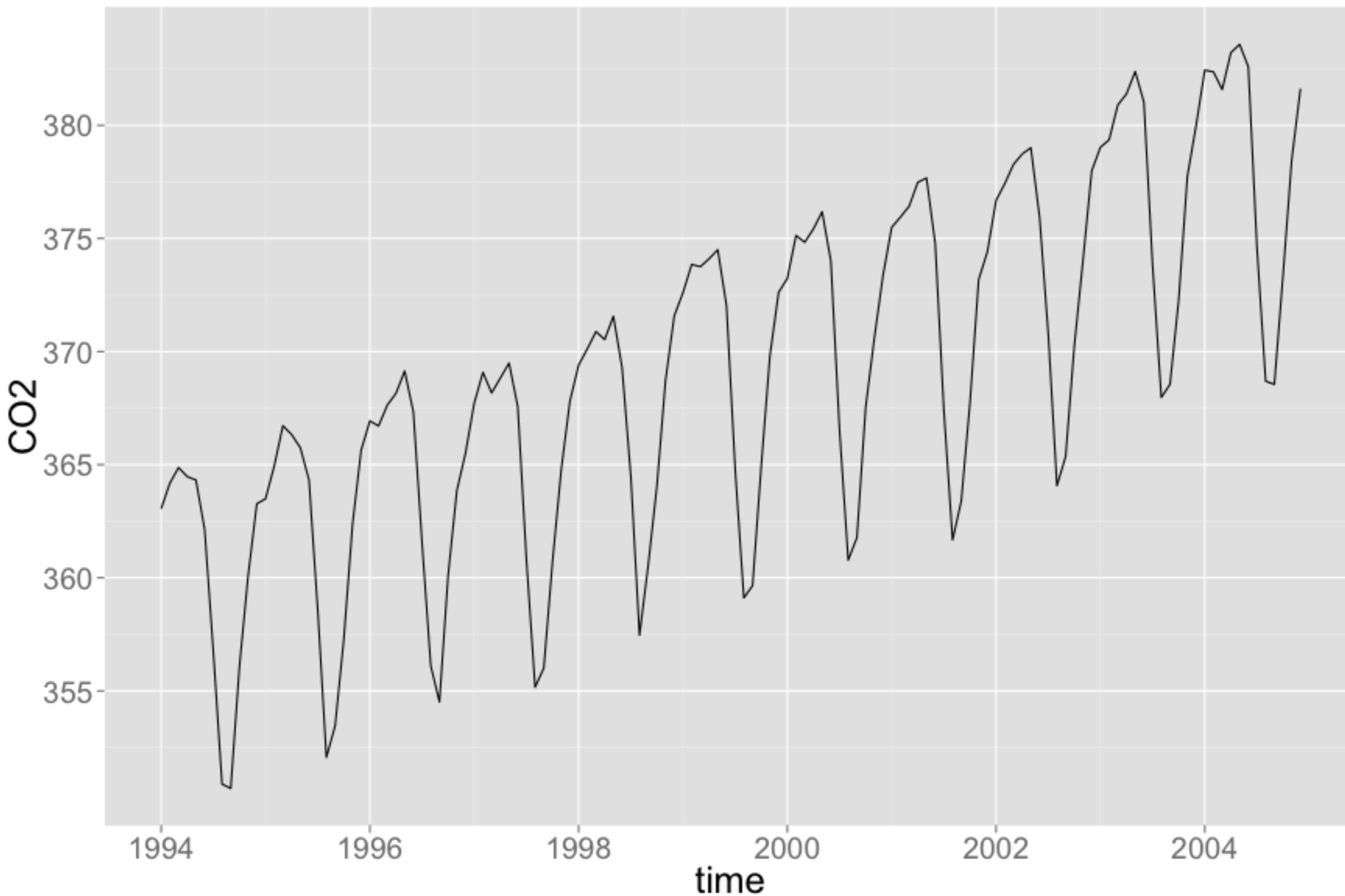
$y_t = \nabla_s x_t = x_t - x_{t-s} = (1 - B^s)x_t,$

we will remove the seasonality.

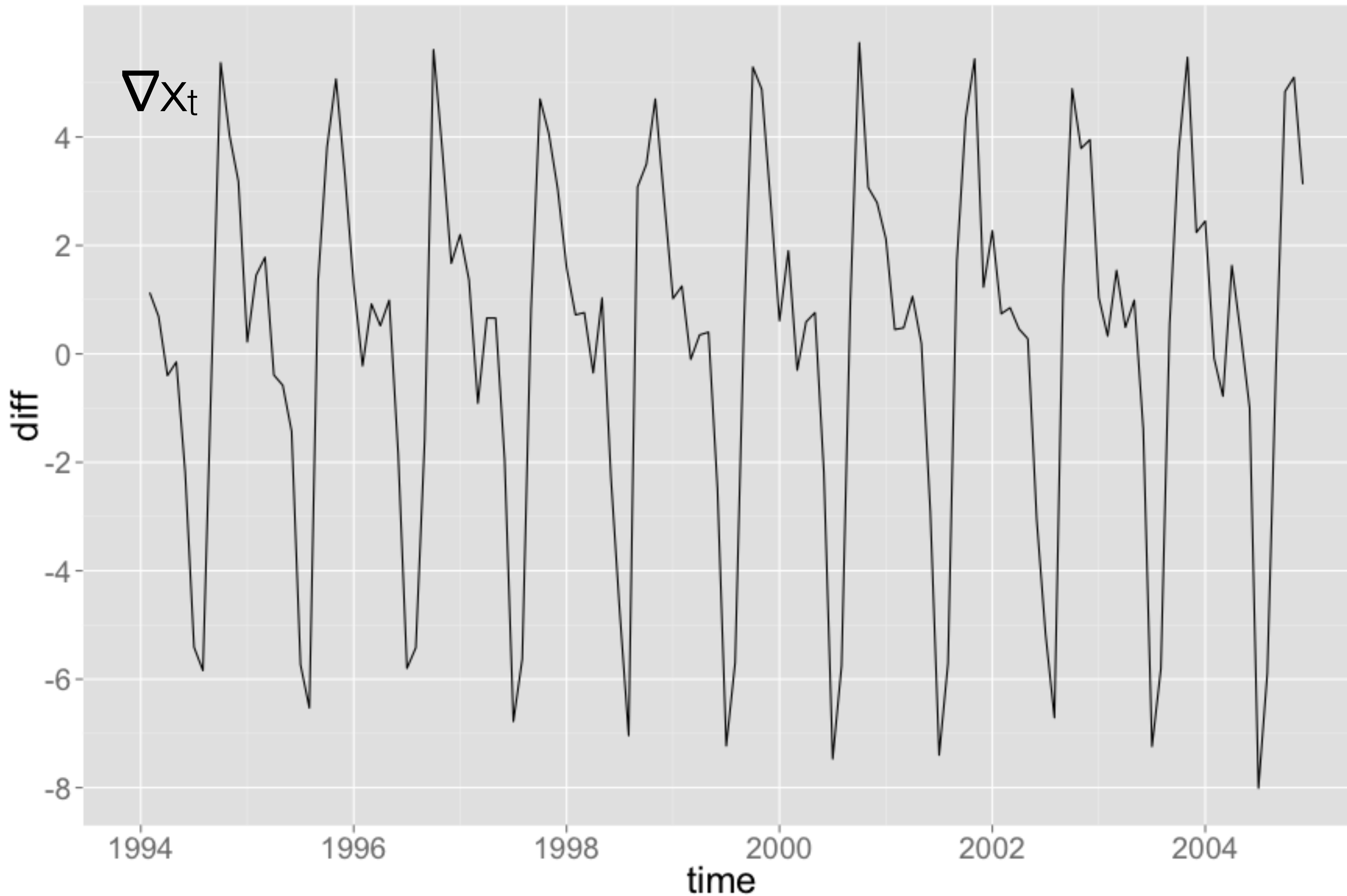
Differencing seasonally D times is denoted,

$\nabla^D_s x_t = (1 - B^s)^D x_t,$

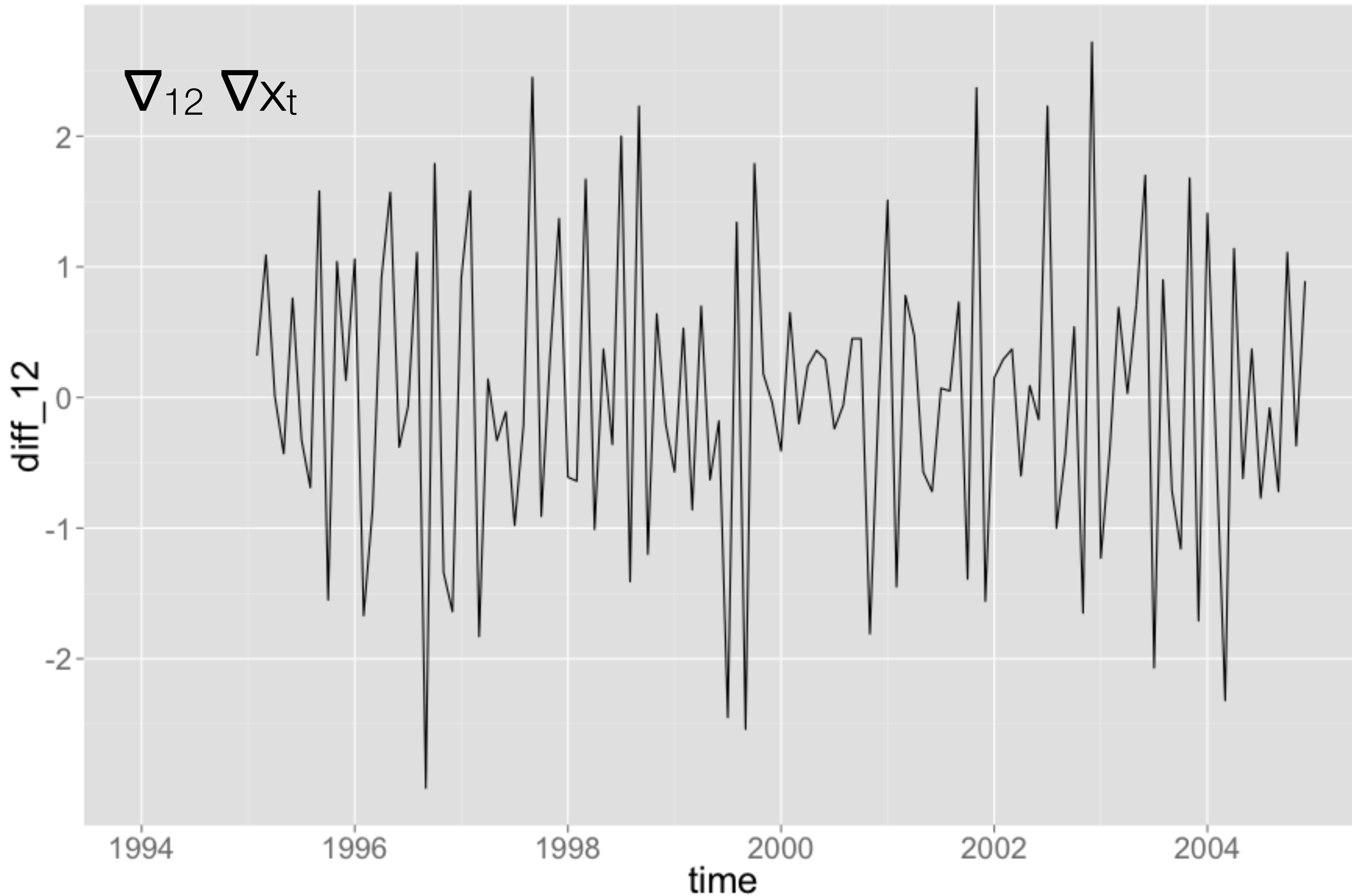
Monthly CO2 level at Alert, Northwest Territories, Canada



First difference



+ first seasonal difference, lag 12



SARIMA

A multiplicative seasonal autoregressive integrated moving average model,

SARIMA(p, d, q) x (P, D, Q)$_s$

is given by

$$\Phi(B^s)\phi(B)\ \nabla^D_s \nabla^d x_t = \Theta(B^s)\theta(B)w_t$$

$\nabla^D_s \nabla^d x_t$ is just an ARMA model with lots of coefficients set to zero.

Have to specify s, then choose p, d, q, P, D and Q

Find model for SARIMA$(1,0,0)\times(0,1,1)_{12}$

Your turn

Find model for $\text{SARIMA}(0,1,1) \times (0,1,1)_{12}$

Procedure for **S**ARIMA modeling

We'll assume the primary goal is getting a forecast.

1. Plot the data. Transform? Outliers? Differencing?

2. Difference to remove trend, find d. Then difference to remove seasonality, find D.

3. Examine acf and pacf of differenced series. Find P and Q first, by examining just at lags s, 2s, 3s, etc. Find p and q by examining between seasonal lags.

4. Fit SARIMA(p, d, q)x$(P, D, Q)_s$ model to original data.

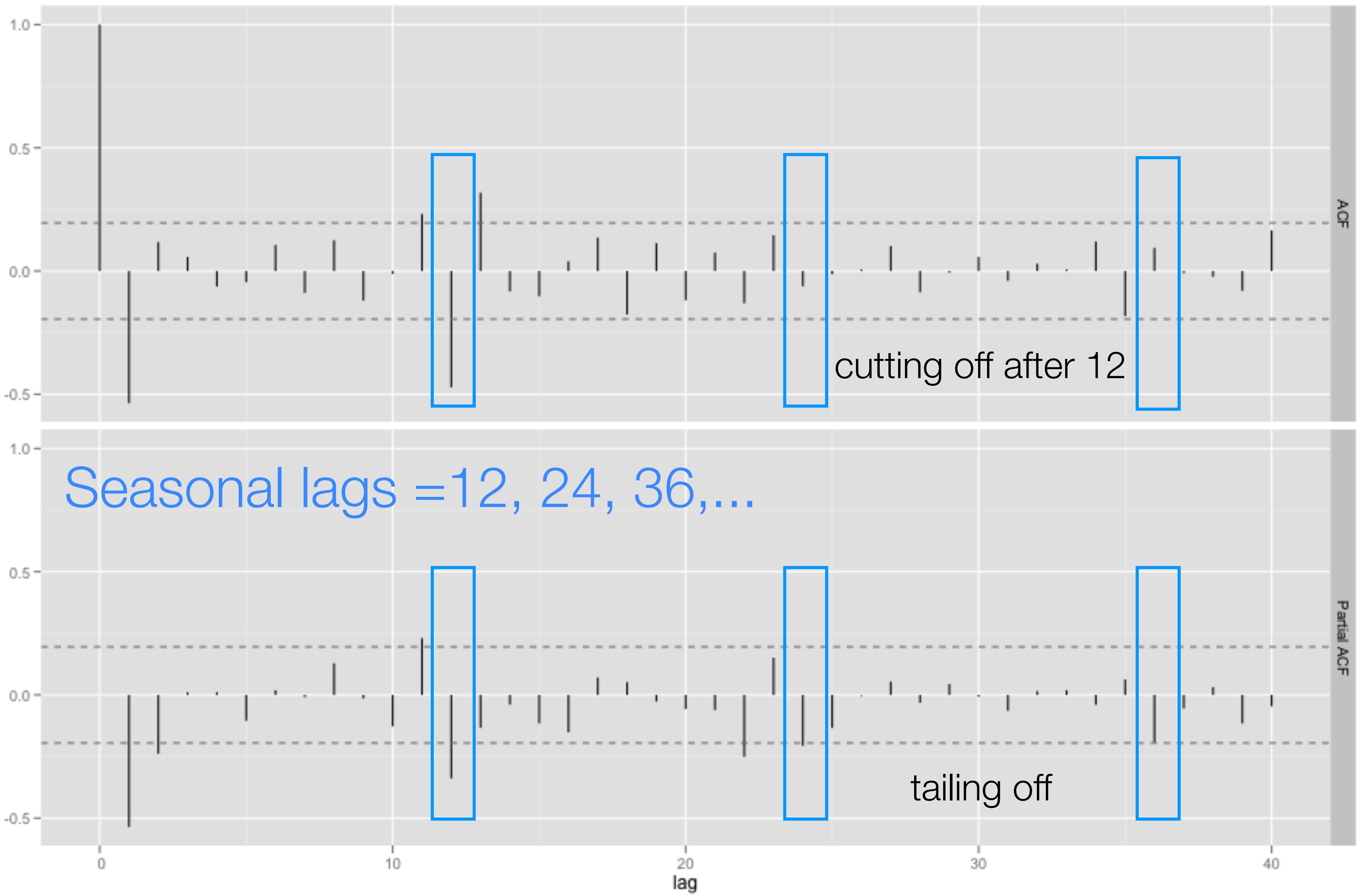
5. Check model diagnostics

6. Forecast (back transform?)

3.

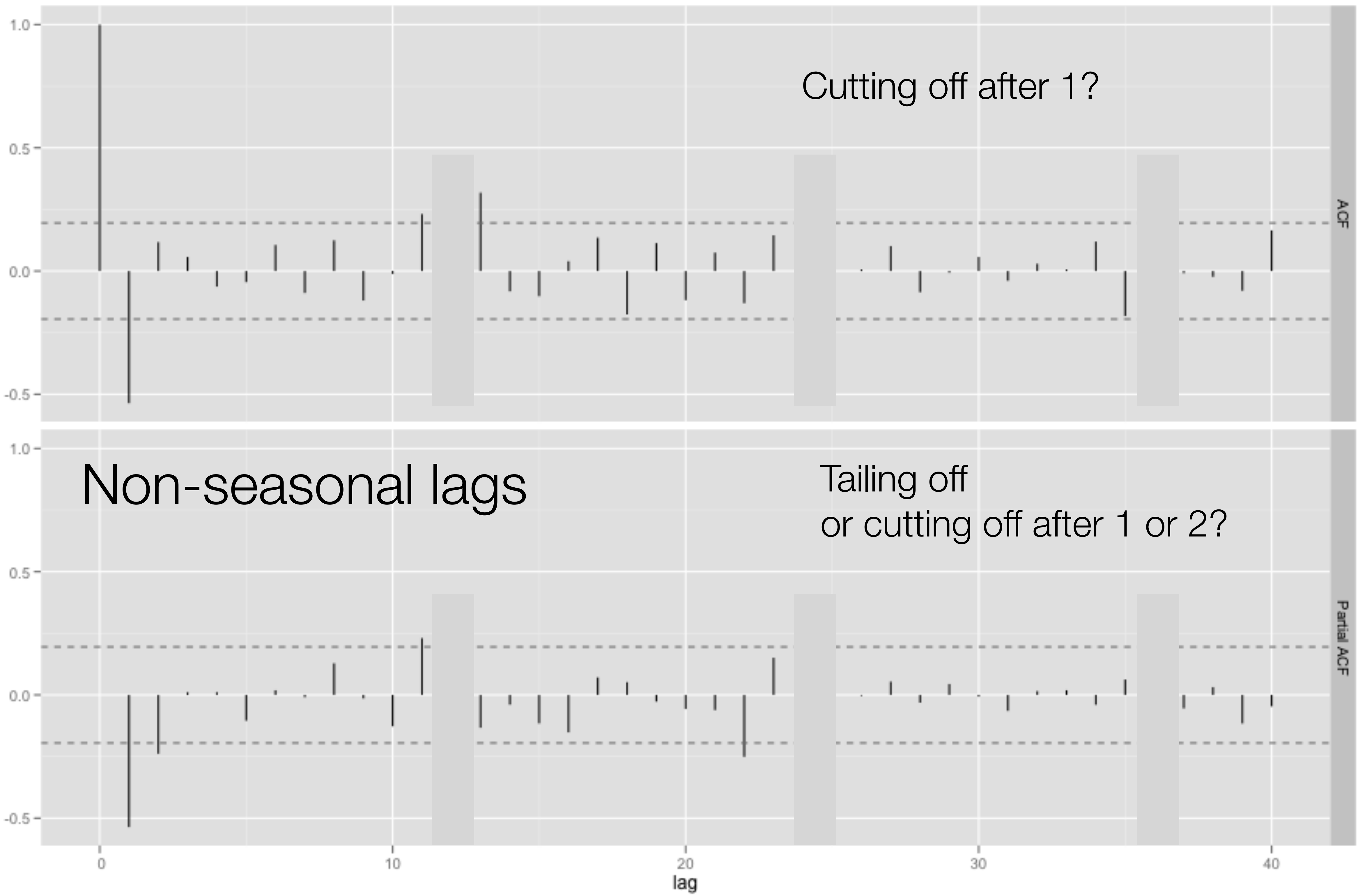
$s = 12, D = 1, d = 1$

ACF & PACF for $\nabla 12\ \nabla x_t$



3.

s = 12, D = 1, d = 1
ACF & PACF for $\nabla 12 \nabla x_t$



4.

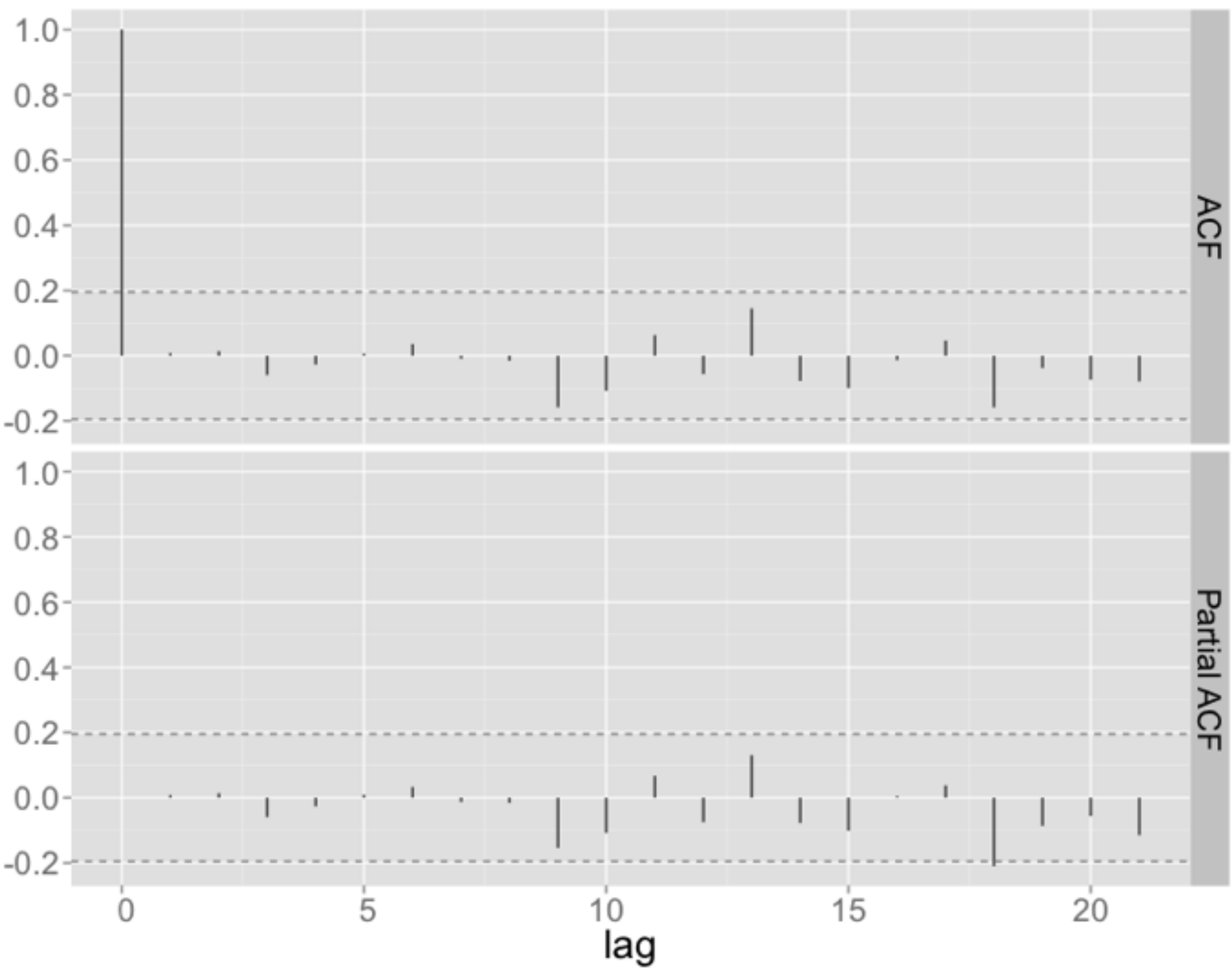
Try

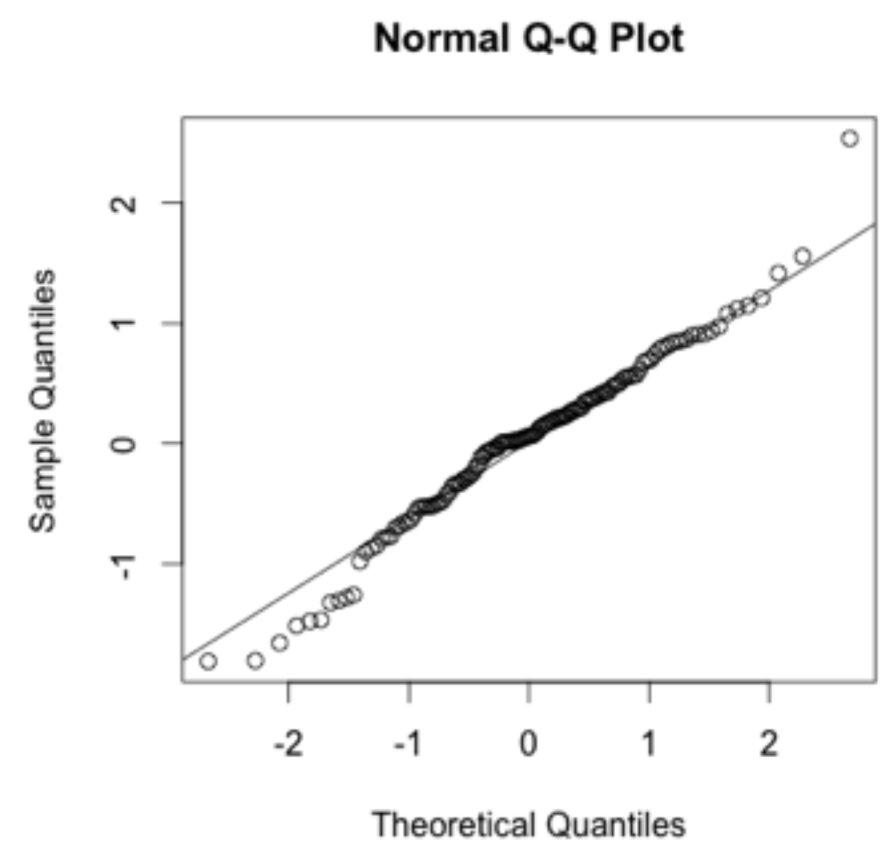
$SARIMA\ (0, 1, 1) \times (0, 1, 1)_{12}$

$SARIMA\ (1, 1, 0) \times (0, 1, 1)_{12}$

$SARIMA\ (1, 1, 1) \times (0, 1, 1)_{12}$

5.


1.0
0.8
0.6
0.4
0.2
0.0
-0.2
ACF
Partial ACF
0
5
10
15
20
lag


Normal Q-Q Plot
Sample Quantiles
Theoretical Quantiles
-2
-1
0
1
2

6.